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Herman Pauwels [a] , Arnout de Meyere [a] & Johan Fornier [a]

[a] Department of Electronics and Information Systems, University of Gent, 41, Sint-Pietersnieuwstraat, B-9000, Gent, Belgium

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ANALYTICAL TREATMENT OF GREY LEVELS IN ANTIFERROELECTRIC LIQUID CRYSTAL DISPLAYS.

HERMAN PAUWELS, ARNOUT DE MEYERE and JOHAN FORNIER.
Department of Electronics and Information Systems,
University of Gent, 41, Sint-Pietersnieuwstraat,
B-9000, Gent, Belgium.

Abstract Under certain simplifying assumptions, which are believed to be non-essential, it is possible to treat grey levels in antiferroelectric liquid crystal displays analytically. This shows more clearly than simulations which parameters play an essential role.

INTRODUCTION

Antiferroelectric liquid crystals have been studied for application in displays[1]. They combine fast switching speed with the possibility to realize grey levels. Recently[2] a prototype display was shown exhibiting grey levels, and thus full colour, at video rates. Various attempts[3] to simulate the most essential features of the device have been reported.

In the present paper, we want to draw the attention to the fact that a very simple analytical model exists that demonstrates very clearly the essential features of the device. It has the usual advantages of an analytical solution over a numerical simulation : showing more clearly the influence of parameter variations and thereby bringing into perspective what is essential in the device.

Some simplifying assumptions must be made : (1) bookshelf structure instead of chevron structure; (2) uniform orientation of the molecules (i.e. the director) throughout the thickness of the display. This last assumption also implies that the anchoring to the alignment layers can only be described by one constant, expressing the preference of the molecules to be parallel

to the glass plates, thereby omitting the polar interaction.

(A) FLC molecules are ordered in layers. In the bookshelf structure these layers are perpendicular to the glass plates and to the rubbing direction of the alignment layers. The molecules lie on a cone with axis perpendicular to the layers and a given cone-angle. The permanent polarization lies in the plane of the layers and perpendicular to the molecules. Its orientation is described by the angle φ as shown in Figure 1. There are two positions on the cone, $\varphi=0$ and $\varphi=\pi$, or "up" and "down" respectively, where the molecules are parallel to the glass, and the polarizations perpendicular to the glass. AFLC are characterized by the fact that in consecutive layers the molecules have a tendency to be at opposite positions on the cone, i.e. their φ-angles differ preferably by π.

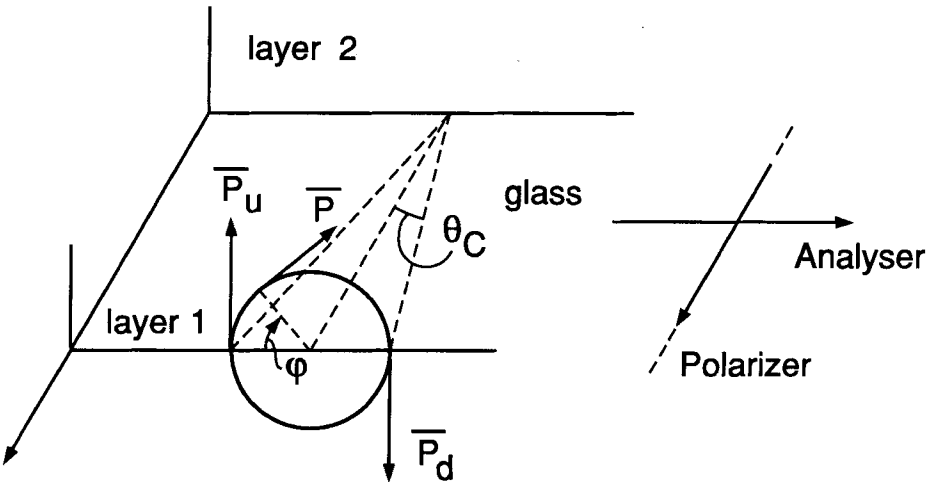

FIGURE 1 Orientation of molecules and polarization in one layer.

STABLE STATES

If φ_1 and φ_2 describe the polarization orientations in consecutive layers as shown in Figure 2, then one

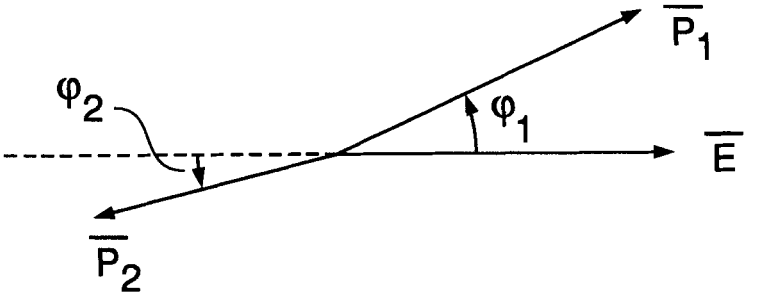

FIGURE 2 Polarization orientations in consecutive layers.

finds stable states in the presence of an electric field E by minimalization of the energy density term.

$$W = -A\cos(\varphi_1 - \varphi_2) - EP\cos\varphi_1 + EP\cos\varphi_2 - \gamma(\cos^2\varphi_1 + \cos^2\varphi_2) \tag{1}$$

The first term expresses the "antiferroelectric" tendency of the polarizations to be antiparallel; it is a term similar to the energy density expression of antiferromagnetism. The second and third term express the tendency of the polarizations to aligh with the electric field. The final term expresses the tendency of the molecules to be parallel to the glass plates. The necessary and sufficient conditions for a state (φ_1, φ_2) to constitute a minimum of W is

$$\frac{\partial W}{\partial \varphi_1} = \frac{\partial W}{\partial \varphi_2} = 0 \tag{2}$$

$$\text{and} \quad \frac{\partial^2 W}{\partial \varphi_1^2} > 0 \quad \text{or} \quad \frac{\partial_2 W}{\partial \varphi_2^2} > 0 \tag{3}$$

$$\text{and} \quad \left(\frac{\partial^2 W}{\partial\varphi_1\partial\varphi_2}\right)^2 < \frac{\partial^2 W}{\partial\varphi_1^2}\,\frac{\partial^2 W}{\partial\varphi_2^2} \tag{4}$$

An easier way to investigate the stability of a (φ_1, φ_2) state, is to verify that W is a minimum along a path away from φ_1, φ_2. This allows a one-dimensional investigation but, of course, gives only a necessary condition for stability.

For sufficently small E-field, one expects the anti-ferroelectric state (AF) with $\varphi_1=0$ and $\varphi_2=0$ to be stable. We therefore investigate W along the path $\varphi_1=0$ and varying φ_2 :

$$\frac{\partial W}{\partial\varphi_2} = A\sin\varphi_2 - EP\sin\varphi_2 + \gamma\sin 2\varphi_2 \tag{5}$$

In order for W to be minimum for $\varphi_2=0$, $\partial W/\partial\varphi_2$ must go from negative, over zero to positive in passing $\varphi_2=0$. By referring the $\gamma\sin 2\varphi_2$ curve with respect to the $(EP-A)\sin\varphi_2$ curve as shown in Figure 3, one finds that the AF-state becomes unstable if $EP > A+2\gamma$ and that then only the $\varphi_2=\pi$, i.e. the ferroelectric up state (FU), remains probably stable. Investigation of equations (2), (3) and (4)

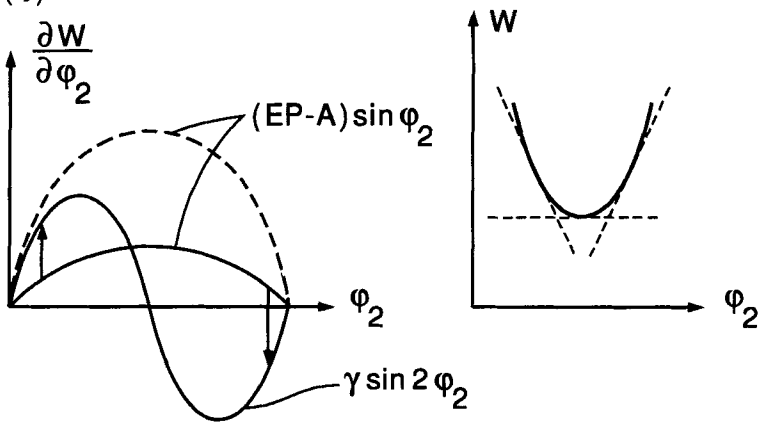

FIGURE 3 Investigation of the AF state along the path $\varphi_1=0$, φ_2 varying. — probably stable situation, --- unstable situation of $\varphi_2=0$.

show that a necessary and sufficient condition for the AF state to be stable is

$$EP < 2\sqrt{\gamma(A+\gamma)} \tag{6}$$

This is shown in the Appendix.

If now for sufficiently large electric field, the AF state has switched to the ferroelectric up state (FU), one can decrease the electric field and wonder when the FU state becomes unstable. For symmetry reasons, this most likely occurs along the path $\varphi_2 = \pi - \varphi_1$, i.e. for the "symmetrical up" state (SU). We therefore investigate

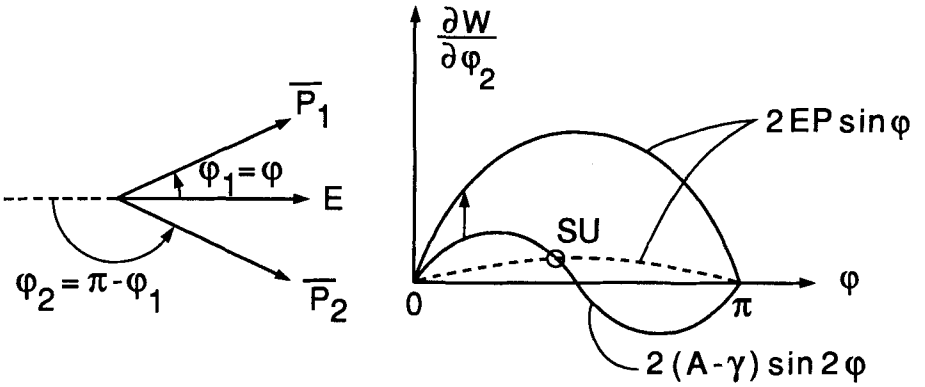

FIGURE 4 Investigation of the FU-state (φ=0) and the symmetrical up state (SU).

$\partial W/\partial\varphi$ along this path.

$$\frac{\partial W}{\partial \varphi} = -2A\sin 2\varphi + 2EP\sin\varphi + 2\gamma\sin 2\varphi \tag{7}$$

By referring the $EP\sin\varphi$ curve with respect to the $(A-\gamma)\sin 2\varphi$ curve as shown on Figure 4, we see that the FU state becomes unstable if

$$EP < 2(A - \gamma) \tag{8}$$

and that then the encircled solution

$$EP\sin\varphi = (A-\gamma)2\sin\varphi\cos\varphi$$

or

$$\cos\varphi = \frac{EP}{2(A-\gamma)} \tag{9}$$

becomes probably stable. A more careful investigation of the conditions (2), (3) and (4) shows that indeed the FU-state is stable as long as EP > 2(A-γ), but that the SU-state only remains stable as long as

$$\cos\varphi > \sqrt{\gamma/(\gamma+A)} \tag{10}$$

For lower values of E the SU state also becomes unstable and the liquid crystal jumps back to the AF-state. This also is shown in the Appendix.

AVERAGE POLARIZATION AND TRANSMISSION.

The polarization averaged over two consecutive layers is zero for the AF-state, is P for the FU state, and is Pcosφ for the SU state. With varying field, the sequence of stable states is thus as shown on Figure 5.

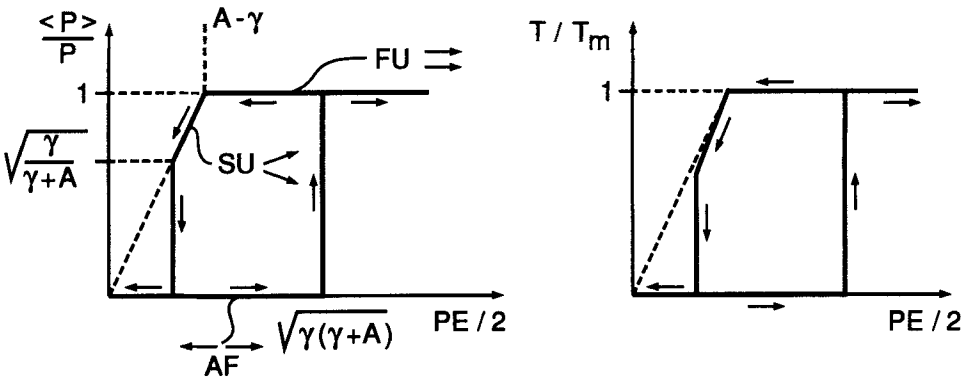

FIGURE 5 Average Polarization (left) and Transmission (right) versus electric field for A/3 < γ < A.

This sequence requires that 0 < A-γ < √γ(γ+A), or A/3 < γ < A. The maximum φ-angle of the SU state then varies between 45° and 60°.

The transmission of the display is determined by the average orientation of the molecules over two consecutive layers. The thickness of the layers is indeed at least one order of magnitude smaller than the wavelength. This average orientation is for the three stable states parallel to the glass plates. For the AF state it is perpendicular to the layers (θ=0), for the FU state it makes an angle θ_c (= cone angle) with this perpendicular direction, and for the SU state, it makes an angle θ_φ given by

$$tg\theta_\varphi = \cos\varphi . tg\theta_c \tag{11}$$

If the polarizator is oriented perpendicular to the layers and the analysator parallel to the layers as shown in Figure 1, the transmission obeys the well known formula

$$T = \sin^2 2\theta \sin^2 \frac{\pi \Delta n d}{\lambda} \tag{12}$$

This is shown in Figure 5, right. In the SU state the transmission decreases somewhat steeper than linear with the electric field.

GREY LEVELS

Grey levels occur when the transition fields between the various stable states vary statistically over the area of the pixel, so that at a given field some parts of the pixel are in one state, some parts in another state. If for instance the anchoring constant γ is uniformly (over the area of the pixel) distributed between values γ_1 and γ_2, we obtain the transmission curves shown in Figure 6. The border line at the right hand side running from

$$E_2 = \frac{2}{P}\sqrt{\gamma_1(\gamma_1+A)} \text{ to } E_3 = \frac{2}{P}\sqrt{\gamma_2(\gamma_2+A)} \tag{13}$$

is described by

$$E = \frac{2}{P}\left[\gamma_1 + (\gamma_2-\gamma_1)\frac{T}{T_m}\right]^{1/2}\left[A+\gamma_1+(\gamma_2-\gamma_1)\frac{T}{T_m}\right]^{1/2} \quad (14)$$

but is hardly discernible from a straight line. The border lines on the left all start from

$$E_1 = \frac{2}{P}(A - \gamma_1) \quad (15)$$

but run to T=0 in a nonlinear, rather complicated way. We did not calculate the actual form, because these curves are in fact not needed. The addressing goes as follows.

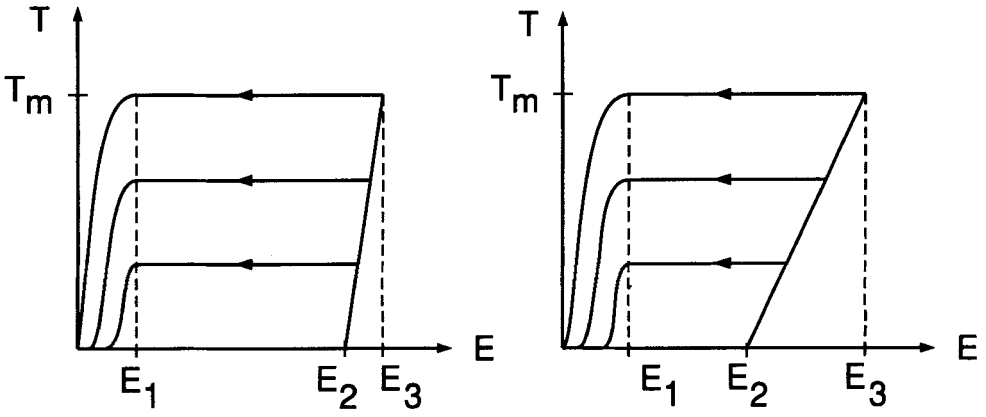

FIGURE 6 Grey level transmission curve. Left is for γ between 3A/4 and A. Right is for γ between 0.56A and A.

During a line addressing time, the line is put at E_2 and the modulating voltages on the columns vary between 0 and E_3-E_2. When the line is not addressed it is put at E_1. The next frame time the opposite voltages are used. In the Figure on the left the modulating voltages are rather small compared to the line addressing voltage, but there is certainly no cross talk. In the figure on the right γ

is uniformly distributed between 0.56A and A. This value is chosen such that $E_2-E_1 = E_3-E_2$. This is the limit which just advoids cross talk, but the modulating voltages are somewhat larger.

CONCLUSION

In this article no technological or theoretical innovations were presented, but it was shown that a very simple model exists that can be treated completely analytically. It required two simple approximations : bookshelf structure and no variation throughout the thickness of the liquid crystal. It was shown that grey levels are possible if the switching fields vary over a pixel area. This can be achieved for instance by varying the anchoring constant over the pixel area. That this can be achieved technologically, was shown in the recent literature[2]. However whether these technological tricks can be controlled sufficiently to lead to an industrial product remains to be proved.

APPENDIX

The AF-state, $\varphi_1=\varphi_2=0$, obeys Eq. (2). The second derivatives are

$$\frac{\partial^2 W}{\partial \varphi_1^2} = A + 2\gamma + EP, \qquad \frac{\partial^2 W}{\partial \varphi_2^2} = A + 2\gamma - EP$$

$$\frac{\partial^2 W}{\partial \varphi_1 \partial \varphi_2} = -A$$

so that condition (4) leads to $EP < 2\,[\gamma(\gamma+A)]^{1/2}$.

The FU state $\varphi_1=0$, $\varphi_2=\pi$, obeys Eq. (2). The second derivatives are

$$\frac{\partial^2 W}{\partial \varphi_1^2} = \frac{\partial^2 W}{\partial \varphi_2^2} = EP + 2\gamma - A, \qquad \frac{\partial^2 W}{\partial \varphi_1 \partial \varphi_2} = A$$

so that again condition (4) leads to $EP > 2(A-\gamma)$.

For the SU state, $\varphi_1=\varphi$ and $\varphi_2=\pi-\varphi$, Eqs. (2) are fulfilled if $\cos\varphi = EP/2(A-\gamma)$. The second derivatives are

$$\frac{\partial^2 W}{\partial \varphi_1^2} = \frac{\partial^2 W}{\partial \varphi_2^2} = EP\cos\varphi + (2\gamma - A)\cos 2\varphi$$

$$\frac{\partial^2 W}{\partial \varphi_1 \partial \varphi_2} = A\cos 2\varphi$$

with

$$\cos 2\varphi = 2\cos^2\varphi - 1 \quad \textit{and} \quad \cos\varphi = EP/2(A-\gamma)$$

The condition (3) then leads to

$$\cos\varphi > \sqrt{\frac{2\gamma - A}{2\gamma}}$$

The condition (4) must be investigated in two steps. If $\cos 2\varphi > 0$ ($\varphi < 45°$), then it is always fulfilled. If $\cos 2\varphi < 0$ ($\varphi > 45°$) then it leads to the condition

$$\cos\varphi > \sqrt{\frac{\gamma}{\gamma + A}}$$

If $\gamma < A$ then the second condition is the most severe.

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